Time, Speed and Distance

TIME, SPEED AND DISTANCE

Speed

The rate at which any moving body covers a particular distance is called its speed.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}};\ \text{Time} = \frac{\text{Distance}}{\text{Speed}};$$

$$\text{Distance} = \text{Speed} \times \text{time}$$

Unit:

SI unit of speed is matre per second (mps). It is also measured in kilometres per hour (kph) or miles per hour (mph).

Basic Conversions :

(i)
- 1 hour = 60 minutes = 60 × 60 seconds.
- 1 km = 1000 m
- 1 km = 0.6214 mile
- 1 mile = 1.609 km i.e. 8 km = 5 miles
- 1 yard = 3 feet
- 1 foot = 12 inches
- 1 km/h = $\frac{5}{18}$ m/sec,
- 1 m/sec = $\frac{18}{5}$ km/h
- 1 miles/hr = $\frac{22}{15}$ ft/sec

Shortcut Approach

- Average speed = $\frac{\text{Total distance}}{\text{Total time}}$
- While travelling a certain distance (d), if a man changes his speed in the ratio m : n, then the ratio of time taken becomes n : m.
- If a certain distance (d), say from A to B, is covered at 'a' km'hr and the same distance is covered again say from B to A to B in 'b' km'hr, then the average speed during the whole journey is given by:

  Average speed = $\left(\frac{2ab}{a+b}\right)$ km/hr

  Also, if t_1 and t_2 is time taken to travel from A to B and B to A respectively, the distance 'd' from A to B is given by:

  $d = (t_1 + t_2)\left(\frac{ab}{a+b}\right)$

  $d = (t_1 - t_2)\left(\frac{ab}{b-a}\right)$

  $d = (b-a)\left(\frac{t_1 t_2}{t_1 - t_2}\right)$
- If first part of the distance is covered at the rate of v_1 in time t_1 and the second part of the distance is covered at the rate of v_2 in time t_2, then the average speed is $\left(\frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}\right)$

Relative Speed

When two bodies are moving in same direction with speeds S_1 and S_2 respectively, their relative speed is the difference of their speeds.

Relative Speed = $S_1 - S_2$, If $S_1 > S_2$

$= S_2 - S_1$, if $S_2 > S_1$

When two bodies are moving in opposite direction with speeds S_1 and S_2 respectively, then their relative speed is the sum of their speeds.

Relative Speed = $S_1 + S_2$

Shortcut Approach

If two persons (or vehicles or trains) start at the same time in opposite directions from two points A and B, and after crossing each other they take x and y hours respectively to complete the journey, then

$$\frac{\text{Speed of first}}{\text{Speed of second}} = \sqrt{\frac{y}{x}}$$

Usual speed: If a man changes his speed to $\frac{a}{b}$ of his usual speed, reaches his destination late/earlier by t minutes then, Usual time = $\dfrac{\text{Change in time}}{\left(\frac{b}{a}-1\right)}$

A man covers a certain distance D. If he moves S_1 speed faster, he would have taken t time less and if he moves S_2 speed slower, he would have taken t time more. The original speed is given by

$$\frac{2\times(S_1 \times S_2)}{S_1 - S_2}$$

If a person with two different speeds U & V cover the same distance, then required distance

$$= \frac{U \times V}{U - V} \times \text{Difference between arrival time}$$

Also, required distance = Total time taken × $\dfrac{U \times V}{U + V}$

- A policemen sees a thief at a distance of d. He starts chasing the thief who is running at a speed of 'a' and policeman is chasing with a speed of 'b' (b>a). In this case, the distance covered by the thief when he is caught by the policeman, is given by $d\left(\frac{a}{b-a}\right)$.
- A man leaves a point A at t_1 and reaches the point B at t_2. Another man leaves the point B at t_3 and reaches the point A at t_4, then they will meet at

$$t_1 + \frac{(t_2 - t_1)(t_4 - t_1)}{(t_2 - t_1) + (t_4 - t_3)}$$

- Relation between time taken with two different modes of transport : $t_{2x} + t_{2y} = 2(t_x + t_y)$
  Where,
  t_x = time when mode of transport *x* is used single way.
  t_y = time when mode of transport *y* is used single way.
  t_{2x} = time when mode of transport *x* is used both ways.
  t_{2y} = time when mode of transport *y* is used both ways.
  See Example: Refer ebook Solved Examples/Ch-9

TRAINS

A train is said to have crossed an object (stationary or moving) only when the last coach of the train crosses the object complete. It implies that the total length of the train has crossed the total length of the object.

Shortcut Approach

- Time taken by a train to cross a pole/a standing man

$$= \frac{\text{Length of train}}{\text{Speed of train}}$$

- Time taken by a train to cross platform/bridge etc. (i.e. a stationary object with some length)

$$= \frac{\text{length of train+length of platform/bridge etc}}{\text{speed of train}}$$

- When two trains with lengths L_1 and L_2 and with speeds S_1 and S_2 respectively, then
  (a) When they are moving in the same direction, time taken by the faster train to cross the slower train

$$= \frac{L_1 + L_2}{\text{difference of their speeds}}$$

(b) When they are moving in the opposite direction, time taken by the trains to cross each other

$$= \frac{L_1 + L_2}{\text{sum of their speeds}}$$

- Suppose two trains or two bodies are moving in the same direction at u km/hr and v km/hr respectively such that u>v, then their relative speed = (u – v) km/hr.
  If their lengths be x km and y km respectively, then time taken by the faster train to cross the slower train (moving in the same direction) = $\left(\frac{x+y}{u-v}\right)$ hrs.
- Suppose two trains or two bodies are moving in opposite directions at u km/hr and v km/hr, then their relative speed = (u + v) km/hr.
  If their lengths be x km & y km, then:
  Time taken to cross each other = $\left(\frac{x+y}{u+v}\right)$ hrs.
- If a man is running at a speed of u m/sec in the same direction in which a train of length L meters is running at a speed v m/sec, then (v – u) m/sec is called the speed of the train relative to man. Then the time taken by the train to cross the man = $\frac{1}{v-u}$ seconds.
- If a man is running at a speed of u m/sec in a direction opposite to that in which a train of length L meters is running with a speed v m/sec, then (u + v) is called the speed of the train relative to man. Then the time taken by the train to cross the man = $\frac{1}{v+u}$ seconds.
- If two trains start at the same time from two points A and B towards each other and after crossing, they take (a) and (b) hours in reaching B and A respectively. Then,
  A's speed : B's speed = $\left(\sqrt{b}:\sqrt{a}\right)$.
- If a train of length L m passes a platform of x m in t_1s, then time taken t_2s by the same train to pass a platform of length y m is given as $t_2 = \left(\frac{L+y}{L+x}\right) t_1$
- From stations P and Q, two trains start moving towards each other with the speeds a and b, respectively. When they meet each other, it is found that one train covers distance d more than that of another train. In such cases, distance between stations P and Q is given as $\left(\frac{a+b}{a-b}\right)\times$d.
- The distance between P and Q is (d) km. A train with (a) km/h starts from station P towards Q and after a difference of (t) hr another train with (b) km/h starts from Q towards station P, then both the trains will meet at a certain point after time T. Then,
  $T = \left(\frac{d \pm tb}{a+b}\right)$
- If second train starts after the first train, then t is taken as positive. If second train starts before the first train, then t is taken as negative.
- The distance between two stations P and Q is d km. A train starts from P towards Q and another train starts from Q towards P at the same time and they meet at a certain point after t h. if train starting from P travels with a speed of x km/h slower or faster than another train, then

(i) Speed of faster train = $\left(\dfrac{d+tx}{2t}\right)$ km/h

(ii) Speed of slower train = $\left(\dfrac{d-tx}{2t}\right)$ km/h

- A train covers distance d between two station P and Q in t_1 h. If the second of train is reduced by (a) km/h, then the same distance will be covered in t_2 h.
- (i) Distance between P and Q is $\left(\dfrac{t_1 t_2}{t_2 - t_1}\right)$ km
- (ii) Speed of the train = $\left(\dfrac{a t_2}{t_2 - t_1}\right)$ km/h

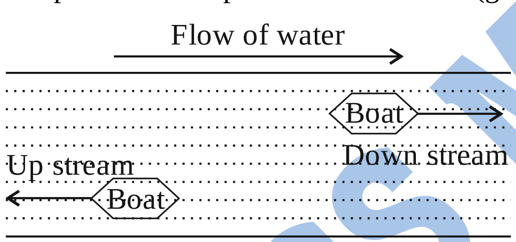
BOATS AND STREAMS

Stream: It implies that the water in the river is moving or flowing.

Upstream: Going against the flow of the river.

Downstream: Going with the flow of the river.

Still water: It implies that the speed of water is zero (generally, in a lake).



Let the speed of a boat (or man) in still water be X m/sec and the speed of the stream (or current) be Y m/sec. Then,

Shortcut Approach

- Speed of boat with the stream (or downstream or D/S)

  = (X + Y) m/sec.

- Speed of boat against the stream (or upstream or U/S)

  = (X – Y) m/sec.

- Speed of boat in still water is

$$X = \frac{(X+Y)+(X-Y)}{2} = \frac{\text{Upstream} + \text{Downstream}}{2}$$

- Speed of the stream or current is

$$Y = \frac{(X+Y)-(X-Y)}{2} = \frac{\text{Downstream} - \text{Upstream}}{2}$$

- A man can row X km/h in still water. If in a stream which is flowing of Y km/h, it takes him Z hours to row to a place and back, the distance between the two places is $\frac{Z(X^2 - Y^2)}{2X}$
- A man rows a certain distance downstream in X hours and returns the same distance in Y hours. If the stream flows at the rate of Z km/h, then the speed of the man in still water is given by $\frac{Z(X+Y)}{Y-X}$ km/hr
- And if speed of man in still water is Z km/h then the speed of stream is given by $\frac{Z(Y-X)}{X+Y}$ km/hr
- If speed of stream is a and a boat (swimmer) takes n times as long to row up as to row down the river, then

  Speed of boat (swimmer) in still water = $\frac{a(n+1)}{(n-1)}$

  Note: *This formula is applicable for equal distances.*
- If a man capable of rowing at the speed (u) m/sec in still water, rows the same distance up and down a stream flow in at a rate of (v) m/sec, then this average speed through the journey is

  $= \frac{\text{Upstream} \times \text{Downstream}}{\text{Man's rate in still water}} = \frac{(u-v)(u+v)}{u}$
- If boat's (swimmer) speed in still water is a km/h and river is flowing with a speed of b km/h, then average speed in going to a certain place and coming back to starting point is given by $\frac{(a+b)(a-b)}{a}$ km/h.